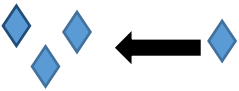
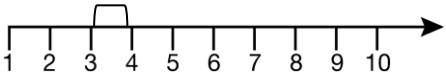
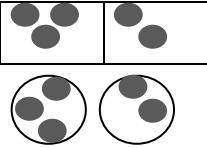
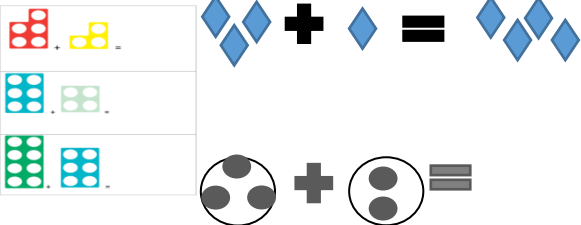
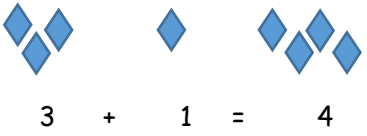
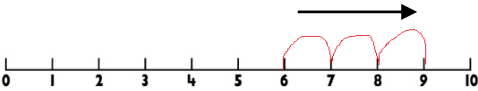
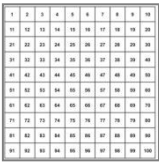
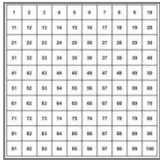
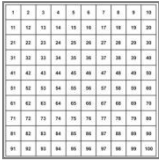
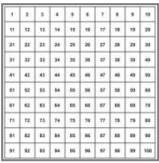


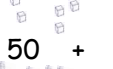


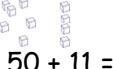
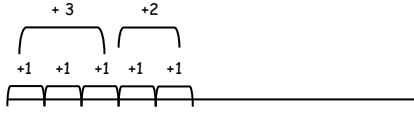
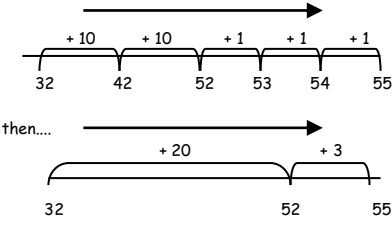




Addition

Stage	Pictorial example	Mathematical language
<p><u>Stage 1</u> Adding 1 more to a group of objects Practically - no recording</p>	 <p>How many now?</p>	<p>How many? 1 more than How many now? How many altogether more/less</p>
<p><u>Stage 2</u> 1 more than a given number using a number line</p>		<p>1 more than How many now?</p>
<p><u>Stage 3</u> Combining two sets of objects practically to find a total (No symbols and no recording)</p>	 <p>Shuffle board Numicon Sorting rings Cubes/objects</p>	<p>plus add makes equals total</p>
<p><u>Stage 4</u> Introduce + = symbols Begin to read number sentences</p>		<p>plus add makes equals total number sentence</p>
<p><u>Stage 5</u> Combining/adding two sets of objects using numbers and symbols. EYFS - sheet 1-3 (Appendix 1)</p>	 <p>3 + 1 = 4</p>	<p>plus add makes equals total number sentence</p>
<p><u>Stage 6</u> Combine two 1 digit numbers using a shuffle board (including number bonds within 10)</p>		<p>plus add makes equals total number sentence how many altogether</p>
<p><u>Stage 7</u> Combine two 1 digit numbers using a number line and then with a 2 digit and a 1 digit number EYFS - sheet 4/5 (Appendix 1)</p>	<p>6 + 3 = 9</p> 	<p>count on how many more how many altogether total answer</p>
<p><u>Stage 8</u> Adding two 1 digit numbers by putting large number in their head and then counting on the smaller number and then a 2 digit and a 1 digit number</p>	<p>4 + 7 =</p> <p>Put 7 in your head, hold up 4 fingers...count on from 7</p>	<p>largest smallest digit count on makes/equals total plus/add</p>

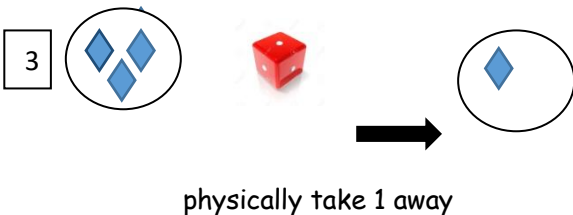
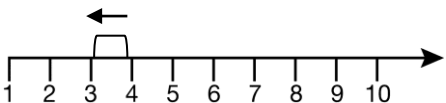

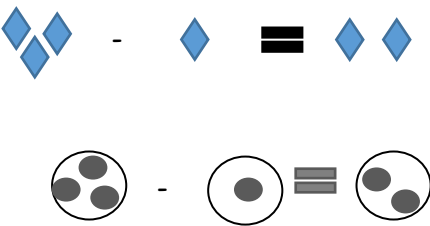
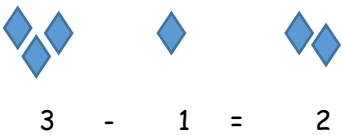
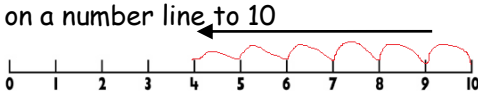

<p><u>Stage 9</u> Find and recall number facts of 4-10. First with objects practically Then begin to record these facts Up to 30, then 50, then 100.</p>	$\begin{array}{ccc} _ + _ = 4 & _ + _ = 5 & _ + _ = 6 \\ _ + _ = 4 & _ + _ = 5 & _ + _ = 6 \\ _ + _ = 4 & _ + _ = 5 & _ + _ = 6 \\ _ + _ = 4 & _ + _ = 5 & _ + _ = 6 \end{array}$ <p>And 7,8,9 and 10 as well</p>	<p>Number facts Addition facts Number bonds total</p>
<p><u>Stage 10</u> Find 1 more than a given number on a hundred square</p>	<p>Find 1 more than 14</p> <p>Find 1 more than 28</p>  <p>(→)</p>	<p>1 more Total 1 more than</p>
<p><u>Stage 11</u> Add a two digit and a 1 digit number using a hundred square. Move onto bridging the next ten (Children know what number comes next once they reach the end of a line)</p>	<p>$22 + 5$ Find 22 on the square, add 5</p> <p>Find 29, add 5 (bridging 10)</p> 	<p>2 digit number 1 digit number Add Plus More Sum of row</p>
<p><u>Stage 12</u> Add a two digit and a ten number using a hundred square. Move onto adding multiples of 10 (e.g. 30, 40)</p>	<p>$28 + 10 =$ ↓ Find 28 on the square, add 10</p> <p>$28 + 20 =$ ↓ ↓ ↓ ↓</p> 	<p>Tens 2 digit number Add Plus More Sum of Multiples of 10</p>
<p><u>Stage 13</u> Add a two digit and a two digit number together using a hundred square. Without bridging , then with bridging 10</p>	<p>$24 + 32 =$</p> <p>Add the tens first ↓ Then the ones →</p> 	<p>Tens Ones Add Plus More Sum of Two digit</p>
<p><u>Stage 14</u> Add by partitioning into tens and ones using -tens and ones rods - practically. -numicon -10p and 1p coins (Add the tens first and then the ones)</p> <p>Then partitioning (bridging ten) Eg. $37 + 24$</p>	<p>$32 + 23 =$</p>  <p>$30 + 20 = 50$ (add the tens)</p>  <p>$2 + 3 = 5$ (add the ones)</p>  <p>$50 + 5 = 55$</p>  <p>$37 + 24$</p> <p>$30 + 20 = 50$ (add the tens)</p>  <p>$7 + 4 = 11$ (add the ones)</p>  <p>$50 + 11 =$</p>	<p>tens ones arrow cards How many tens? How many ones? partitioning sum of add, plus total, altogether</p>

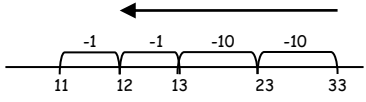
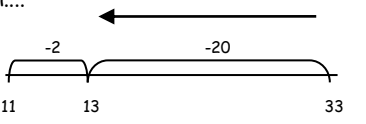
<p><u>Stage 15</u> Add three 1 digit numbers Using blank number line - mentally</p>	<p>$4 + 3 + 2 =$</p> 	<p>add plus more greater altogether total sum of</p>
<p><u>Stage 16</u> Add by partitioning into tens and ones (recording only - no objects) (Add the tens first and then the ones)</p>	<p>$32 + 23 = 55$ $30 + 20 = 50$ (add the tens) $2 + 3 = 5$ (add the ones) $50 + 5 = 55$ (add the ones and tens)</p>	<p>tens ones arrow cards How many tens?ones? partitioning add, plus sum of total, altogether</p>
<p><u>Stage 17</u> Adding by partitioning using a blank number line (Add the tens first and then the ones)</p>	<p>$32 + 23 =$</p> 	<p>tens ones arrow cards How many tens?ones? partitioning blank number line add, plus sum of total, altogether</p>
<p><u>Stage 18</u> Add by using the expanded column method (Add the ones first and then the tens)</p>	$\begin{array}{r} 32 \\ + 23 \\ \hline 5 \quad (2 + 3) \\ \underline{50} \quad (30 + 20) \\ 55 \end{array}$	<p>tens ones arrow cards How many tens? How many ones? partitioning expanded column method add, plus sum of total, altogether</p>
<p><u>Stage 19</u> Add using the compact column method (no expanding)</p>	$\begin{array}{r} 32 \\ + 23 \\ \hline 55 \end{array}$	<p>tens ones arrow cards How many tens? How many ones? partitioning compact column method add, plus sum of total, altogether</p>
<p><u>Stage 18</u> Addition number sentences with missing numbers in all possible places</p>	$\begin{array}{ll} 3 + 4 = \square & \square = 3 + 4 \\ 3 + \square = 7 & 7 = \square + 4 \\ \square + 4 = 7 & 7 = 3 + \square \\ \square + \nabla = 7 & 7 = \square + \nabla \end{array}$	<p>missing number How many more? How many more to make? 3 plus how many more to make...?</p>

<p><u>Stage 9</u> Adding two single digits. Equals sign should be written either side of number sentence to show that the sign is not just interpreted as 'the answer'</p>	$5 = 4 + 1$ $4 + 1 = 3 + 2$ $4 + 1 = 5$ $1 + 1 + 1 + 1 + 1 = 5$	<p>equals balances same as</p>
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<p><u>Notes for Addition</u> For deep mastery:</p> <ul style="list-style-type: none"> * Link to worded one step problems, building up to two step problems. * Use addition within problem solving/worded problems and make connections to measures (time, length, weight, scales) and money. * Addition is commutative (2 numbers can be added in any order) * Practise addition and subtraction to 20 to become fluent in deriving facts such as $3 + 7 = 10$, $10 - 7 = 3$ and $7 = 10 - 3$ to calculate; $30 + 70 = 100$, $100 - 70 = 30$ and $70 = 100 - 30$ to ensure they are checking their calculations by adding to check subtraction and adding numbers in a different order to check addition $5 + 2 = 7$, $2 + 5 = 7$
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Subtraction

Stage	Pictorial example	Mathematical language
<p><u>Stage 1</u> Singing number songs and action rhymes</p>	<p>5 little speckled frogs 10 in the bed 5 little men on a flying saucer 5 currant buns 10 green bottles</p>	<p>1 less take away How many are left? smaller</p>
<p><u>Stage 2</u> Taking away 1 from a group of objects Use a number card to place a given amount of objects into a circle. Roll a dice with 1 spots on it Practical - no recording</p>	 <p style="text-align: center;">physically take 1 away</p>	<p>How many left? 1 less than 1 less smaller How many now? How many altogether number words take away</p>
<p><u>Stage 3</u> 1 less than a given number using a number line</p>		<p>1 less than How many now? takeaway count back</p>
<p><u>Stage 4</u> Taking away a given amount from a group of objects. No recording.</p>		<p>How many are left? Takeaway</p>
<p><u>Stage 5</u> Introduce - = symbols Begin to read number sentences Practical - no recording at first Build up to recording (see subtraction sheets 1-4)</p>		<p>subtract less take away makes How many are left? number sentence</p>
<p><u>Stage 6</u> Subtracting 1 set from another set of objects using numbers and symbols. Recording EYFS - sheet 1-4 (Appendix 2)</p>	 <p style="text-align: center;">$3 - 1 = 2$</p>	<p>subtract less take away makes How many are left? number sentence</p>
<p><u>Stage 7</u> Subtracting a 1 digit number using a number line from a 1 digit number then a 2 digit number Counting back on a number line (For finding the difference children can also count forwards on the line)</p>	<p>a) $10 - 6 = 4$ Do practically with objects b) Do on a number line to 10  c) Do on a number line to 20</p>	<p>subtract count backwards take away minus difference between How many left?</p>
<p><u>Stage 8</u> Subtracting a 1 digit from a 2 digit number on a hundred square -Up to 30, then 50, then 100. move onto subtracting a 2 digit from a 2 digit number using a hundred square (Subtract the tens first then ones)</p>	 <p>$26 - 24$ Find 26 on the square, subtract 4 by counting backwards</p> <p>$26 - 24$ First subtract the 2 tens (↑) Then subtract the 2 ones (←)</p>	<p>subtract count backwards count on from take away minus difference between How many left? missing numbers hundred square</p>


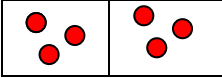
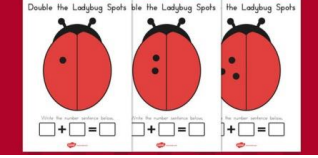
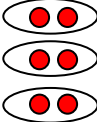
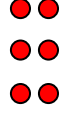

<p><u>Stage 9</u> Subtracting using a blank number line - partitioning method</p> <p>(Subtract the tens first, then the ones)</p>	<p>$33 - 22 =$</p>  <p>then....</p> 	<p>subtract minus less fewer take away minus difference between How many left? tens ones number line</p>
<p><u>Stage 10</u> Subtracting using the expanded column method (Subtract the ones first and then the tens)</p>	$\begin{array}{r} 33 \\ - 22 \\ \hline 1 \quad (3 - 2) \\ \underline{10} \quad (30 - 20) \\ 11 \end{array}$	<p>subtract minus less fewer take away minus difference between How many left? tens ones expanded column method</p>
<p><u>Stage 11</u> Subtract using the compact column method (no expanding)</p>	$\begin{array}{r} 33 \\ - 22 \\ \hline 11 \end{array}$	<p>subtract minus less fewer take away minus difference between How many left? tens ones column method</p>
<p><u>Stage 12</u> Subtraction number sentences with missing numbers in all possible places</p>	$\begin{array}{ll} 7 - 3 = \square & \square = 7 - 3 \\ 7 - \square = 4 & 4 = \square - 3 \\ \square - 3 = 4 & 4 = 7 - \square \\ \square - \nabla = 4 & 4 = \square - \nabla \end{array}$	<p>subtract count backwards count on from take away minus difference between How many left? missing numbers</p>
<p><u>Stage 13</u> Subtract exchanging using two digit numbers</p> <p>Two step column method to ensure children's understanding of the value of the digits (Subtract the ones first then the tens)</p>	<p>$75 - 37 =$</p> $\begin{array}{r} 70 \quad 5 \\ - 30 \quad 7 \\ \hline 30 \quad 8 \end{array}$ <p>(70 - 10) (10 + 5)</p> $\begin{array}{r} 60 \quad 15 \\ \underline{- 30 \quad 7} \\ 30 \quad 8 \end{array}$ $\begin{array}{r} 6 \quad 15 \\ \underline{- 3 \quad 7} \\ 3 \quad 8 \end{array}$	<p>subtract minus less fewer take away minus difference between How many left? tens ones column method exchange</p>

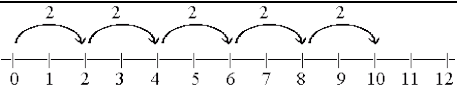
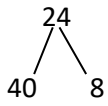
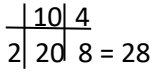
Notes for Subtraction

For deep mastery:

- * Link to worded one step problems, building up to two step problems.
- * Use subtraction within problem solving/worded problems and make connections to measures (time, length, weight, scales) and money.
- * Subtraction is not commutative (2 numbers cannot be subtracted in any order)
- * Recognise the inverse relationship between addition and subtraction and use this to check calculations and missing number problems.
- * Practise addition and subtraction to 20 to become fluent in deriving facts such as
 $3 + 7 = 10$, $10 - 7 = 3$ and $7 = 10 - 3$ to calculate;
 $30 + 70 = 100$, $100 - 70 = 30$ and $70 = 100 - 30$

Multiplication

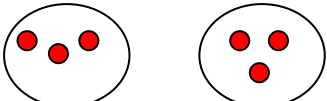
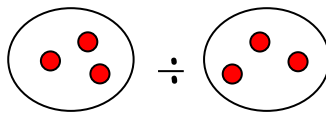
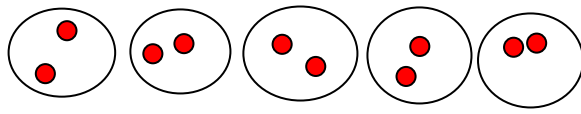
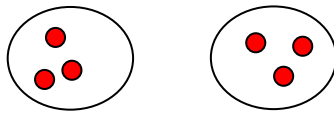
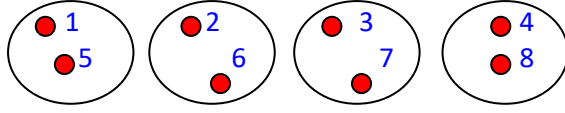
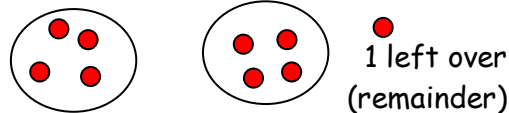
Stage	Pictorial example	Mathematical language
<u>Stage 1</u> Grouping objects into pairs and counting in 2's		counting in 2's, 5's, 10's groups of sets of lots of pairs
<u>Stage 2</u> Counting up in 2's, 5's and 10's with/without actions -Up to 10 times the number (EYFS) -Up to 12 times the number (KS1) Practical - no recording	2 4 6 8 10 12 14 16 18 20 22 24 5 10 15 20 25 30 35 40 45 50 55 60 10 20 30 40 50 60 70 80 90 100 110 120	2's 5's 10's count up in steps of...
<u>Stage 3</u> Doubling using objects	 double 3	double doubling
<u>Stage 4</u> Doubling using objects Recording using pictorial representations and recording using symbols		doubles/eg and add makes/equals
<u>Stage 5</u> Repeated addition with objects	$\begin{matrix} \bullet & + & \bullet & + & \bullet & = \\ \bullet & & \bullet & & \bullet & \end{matrix}$ $2 + 2 + 2 =$ $3 \text{ lots of } 2 =$	repeated addition lots of sets of groups of
<u>Stage 6</u> Using arrays and introduce the X sign	 $2 + 2 + 2 =$ $3 \text{ lots of } 2 =$ $3 \times 2 =$	multiply times lots of groups of sets of arrays
<u>Stage 7</u> Showing that multiplication is commutative (Can be done in any order)	$3 \times 2 = 6$  $2 \times 3 = 6$ 	Inverse arrays multiplication times arrays sets of/groups of/ lots of
<u>Stage 8</u> Repeated addition/multiplication on a number line Show that multiplication is commutative (Can be done in any order)	$2 + 2 + 2 + 2 + 2 = 10$ $5 \text{ lots of } 2 = 10$ $5 \times 2 = 10$ $2 \times 5 = 10$	multiply times lots of groups of sets of arrays

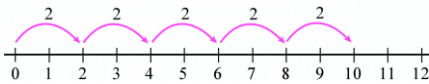
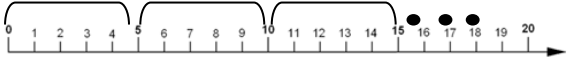

		
<u>Stage 9</u> Doubling a 2 digit number by partitioning		Doubling tens / ones
<u>Stage 10</u> Multiplying a 2 digit and a 1 digit number using the grid method	$2 \times 14 =$ 	grid method times by the tens times by the ones multiplied
<u>Stage 11</u> X = signs and missing numbers	$7 \times 2 = \square$ $\square = 2 \times 7$ $7 \times \square = 14$ $14 = \square \times 7$ $\square \times 2 = 14$ $14 = 2 \times \square$ $\square \times \square = 14$ $14 = \square \times \square$	Missing numbers times multiply
<u>Stage 12</u> Link multiplication facts and division facts Recognise the inverse relationship between multiplication and division and use this to check calculations and missing number problems.	$2 \times 3 = 6$ $3 \times 2 = 6$ $6 \div 3 = 2$ $6 \div 2 = 3$	Inverse relationship multiplication division
<u>Notes for Multiplication</u> For deep mastery: * Link to worded one step problems, building up to two step problems. * Use multiplication within problem solving/worded problems and make connections to measures (time, length, weight, scales) and money. * Multiplication is commutative (2 numbers can be multiplied in any order) * Practise multiplication to become fluent. Connect the 10 multiplication table to place value and the 5 multiplication table to the divisions on the clock face. * Work with a range of materials and contexts in which multiplication relate to grouping quantities, relating to fractions and measures		

****Children need to be able to fluently recall multiplication facts by the end of Key stage 1 for the 2,3,5 and 10 times tables up to 12 times the number e.g. $12 \times 2 = 24$ ****

(Fluency is recalling facts from memory and not using fingers or any resources.)

Division

Stage	Pictorial example	Mathematical language
<p><u>Stage 1</u> Practical sharing out objects up to any amount (1-1 correspondence) Share out 1 for me, 1 for you - repeat until they've all gone. Link to halving</p>	<p>6 shared equally between 2</p> 	<p>sharing divide halving equally the same as</p>
<p><u>Stage 2</u> Practical sharing out of objects between 2 people/sorting rings Share out 1 for me, 1 for you - repeat until they've all gone. Link to halving Introduce the sign for division \div</p>	<p>$6 \div 2 = 3$</p> 	<p>sharing divide halving equally the same as shared by</p>
<p><u>Stage 3</u> Dividing between 2, 3, 5 and 10 groups using circle and dot method</p>	<p>$10 \div 5 =$ $-$</p>  <p>As you count to the total number of objects to be divided put a dot in each circle in turn.</p>	<p>sharing divide halving equally the same as half</p>
<p><u>Stage 4</u> As in stage 2 but do with worded problems Dot and circle method Introduce $\frac{1}{2}$ sign</p>	<p>$6 \div 2 = 3$</p>  <p>I have 6 pieces of lego. If I share them between me and my friend, how many pieces of lego will we get each?</p>	<p>sharing divide halving equally the same as half</p>
<p><u>Stage 5</u> Practical sharing out of objects between 4 people/sorting rings Share out 1 for me, 1 for you - repeat until they've all gone. Dot and circle method too Link to quarters Introduce $\frac{1}{4}$ sign link to quarterly intervals on a clock and quarter turns</p>		<p>sharing divide quarters equally the same as half</p>
<p><u>Stage 6</u> Practical sharing out of objects between 4 people/sorting rings Share out 1 for me, 1 for you - repeat until they've all gone. Dot and circle method too With remainders</p>	<p>$9 \div 2$</p>  <p>1 left over (remainder)</p>	<p>sharing divide shared by not equal remainder left over</p>

<p><u>Stage 7</u> Division on a number line</p>	<p>$10 \div 2 = 5$</p> 	<p>count in 2's counting up number line sharing dividing How many 2's in.....?</p>
<p><u>Stage 8</u> Division on a number line with remainders</p>	<p>$18 : 5 = 3r3$</p> 	<p>count ins counting up sharing dividing How many ...'s in.....? remainder How many left?</p>
<p><u>Stage 9</u> Divide use problem solving on a number line. How many groups of 4 do you need until you get to 12?</p>		<p>Groups of Sharing Dividing Count up in steps of...</p>
<p><u>Stage 10</u> Link multiplication facts and division facts Recognise the inverse relationship between multiplication and division and use this to check calculations and missing number problems.</p>	<p>$2 \times 3 = 6$ $2 \times 2 = 6$ $6 \div 3 = 2$ $6 \div 2 = 3$</p>	<p>Inverse relationship multiplication division</p>
<p><u>Stage 11</u> \div = signs and missing numbers</p>	<p>$6 \div 2 = \square$ $\square = 6 \div 2$ $6 \div \square = 3$ $3 = 6 \div \square$ $\square \div 2 = 3$ $3 = \square \div 2$ $\square \div \triangle = 3$ $3 = \square \div \triangle$</p>	<p>count ins sharing dividing How many ...'s in.....? How many left? missing numbers</p>
<p><u>Stage 12</u> Division using the expanded bus stop method</p>	$\begin{array}{r} 2 \overline{)46} \\ \underline{20} \quad (40 \div 2) \\ \quad \underline{3} \quad (6 \div 2) \\ \quad \quad \underline{23} \end{array}$	<p>sharing dividing How many ...'s in.....? remainder How many left? expanded bus stop</p>
<p><u>Stage 13</u> Division using the bus stop method number</p>	$\begin{array}{r} \quad 23 \\ 2 \overline{)46} \\ \quad \quad - \end{array}$	<p>count ins sharing dividing How many ...'s in.....? remainder How many left? bus stop</p>

Notes for Division

For deep mastery:

- * Link to worded one step problems, building up to two step problems.
- * Use division within problem solving/worded problems and make connections to measures (time, length, weight, scales) and money. ($40 : 2 = 20$, 20 is half of 40)
- * Division is not commutative (2 numbers cannot be divided in any order)
- * Practise division to become fluent. Connect the 5 multiplication table to the divisions on the clock face.

Work with a range of materials and contexts in which division relates to sharing quantities, relating to fractions and measures